



ICE FORCES AGAINST ARCTIC OFFSHORE STRUCTURES

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## I. BACKGROUND

This third quarterly report covers progress on the subject contract from June 1, 1982 to September 1, 1982.

The objective of this research program is to determine the lateral forces on artificial islands and offshore structures which are subject to moving sea ice. This is the major factor governing the design of offshore facilities for petroleum production in the Beaufort, Chukchi, and Bering Seas, a frontier province which encompasses some 262 million acres with a risked mean oil equivalent of 30.8 billion barrels.

The approach taken is to measure the internal ice stress at relatively large distances from such islands, to measure the ice displacement simultaneously, and to determine the effective island width during ridgebuilding events. These events, which fracture the ice adjacent to the islands and structures, represent those time intervals when maximum total forces may be exerted on such man-made structures. They represent the extreme lateral force design condition for the ice thickness and type at that time. Although very high local forces may disturb the gravel or rock slopes of artificial islands, this can be repaired. The more significant issue is whether the lateral resistance to movement of the entire artificial island or offshore structure is sufficient to withstand the maximum total force exerted by the moving ice. Allowance must be made for the thickest ice and the highest velocity of ice movement expected during the operating life of the production facility. A detailed discussion of current practice in such designs was given in the first quarterly report.

In the second quarterly report, the completion and calibration of the electronic data telemetry system was described. The theory necessary for converting gauge output information into principal stress magnitude and direction was also developed during the second quarter, and detailed in that report. The calibration program for gauges frozen into ice blocks was begun, and the experimental determination of the stress concentration factor  $\alpha(\theta)$  for uniaxial stiff gauges was begun.

## II. EXPERIMENTAL PROGRAM

During the third quarter, it appeared that the equipment which had been borrowed from an earlier project (the OCSEAP [MMS] seismic project RU 483), which was terminating, might be required for another seismic project and might not be made available to us for the 1982-83 winter season. Some attention was given to other system alternatives, and also to the costs of acquisition of new equipment. Fortunately, however, at this time, it has been definitely determined that the equipment in question will be made available to us for the 1982-83 winter season, as originally hoped, thus potentially saving an equipment investment of some \$25,000, and implying that the electronic data telemetry system, as built, checked and calibrated in the first quarter, will be ready for field deployment in the coming 1982-83 winter. An additional channel is being added to accommodate one more strain sensor which was determined to be necessary.

The ice movement station consideration has made progress during the third quarter. Quotes in the range of \$40,000 to \$63,000 were received from the electronic distance-measuring equipment vendors; this type of

system is only sufficiently precise if located with two fixed shore stations several kilometers apart, as discussed in our earlier report. The opportunity to observe dynamic annual and perhaps multi-year ice in a winter depth of 31 meters adjacent to the new Dome Petroleum Ltd. artificial structure Uviluk was considered to be greater than the possible movements at the Shell Seal Island location in the Alaskan Beaufort Sea. Arrangements are being made with Dome Petroleum for field deployment there. This isolated site is not within a line-of-sight distance to other fixed reference points, so it is not appropriate to use an electronic distance-measuring system. It was decided to rely upon the original concept which has been used in land-fast ice for nearly a decade --- the wire line system referencing to an anchor point on the seafloor. It should be recognized that the main limitation for this approach is the dynamic range - movements of more than about 130 meters will require manual extension of the cable, and of more than 450 meters, the repositioning of the anchor, depending upon direction. Nevertheless, it is expected that considerable data would be obtained before repositioning will be necessary, and it should be possible to do so in the field later in the season. This system is presently being fabricated, with several improvements over earlier designs. Testing should take place in November 1982. Use of this technique results in appreciable cost savings compared with the microwave ranging system approach.

Calibration of uniaxial gauges and ring strain gauges embedded in ice blocks has continued, and it is now felt that this should be viewed as a longer-term sequence of loading experiments to be continued at a modest level of effort throughout the winter months. Dr. Jerome Johnson has accepted responsibility for this part of the program; he has conduc-

ted similar tests on cylindrical stress gauges and is very experienced in such measurements and their interpretations.

### III. THEORETICAL ANALYSIS

A method of calculating stresses on an artificial island or structure, from the type of data which will be obtained from field measurements, has been presented in a paper by Dr. Jerome B. Johnson at the Symposium on Applied Glaciology, Hanover, NH, in August 1982. This paper, entitled, "A Surface Integral Method for Calculating Ice Loads on Offshore Structures from In-Situ Measurements", is presented in Appendix I of this report, and will be published in due course in the *Journal of Glaciology* as part of the Symposium Proceedings.

### IV. PLANS FOR NEXT QUARTER

Field deployment of the ice stress sensor system remains as the subject of primary emphasis of the project, and is expected to take place at the Dome Petroleum Ltd. Uviluk site either in mid-December 1982 or in early January 1983, depending upon ice thickness. A minimum annual ice thickness of 75 cm. is needed to install the uniaxial gauges and interpret their outputs.

APPENDIX I

A SURFACE INTEGRAL METHOD FOR CALCULATING ICE LOADS  
ON OFFSHORE STRUCTURES FROM IN-SITU MEASUREMENTS

by

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# A SURFACE INTEGRAL METHOD FOR CALCULATING ICE LOADS ON OFFSHORE STRUCTURES FROM IN-SITU MEASUREMENTS

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## ABSTRACT

Two methods are presented for calculating ice loads on structures using measurements from sensors imbedded in an ice sheet and from instruments attached to a structure. The first method uses a mathematical model describing ice/structure interaction for a cylindrical structure to interpret stress measurements. This technique requires only a few sensors to develop an estimate of ice loads. However, analytical and experimental results indicate that using a mathematical model to interpret stress measurements can result in inaccurate load estimates due to uncertainty in the accuracy of the model and the uncertainty of using local ice stresses to calculate total ice forces. The second method of calculating ice loads on structures utilizes Euler and Cauchy's stress principle. The force acting on a structure is determined by summing the stress vectors acting on a surface which encompasses the structure (surface integral method). Application of this technique requires that the shear and normal components of stress be known along the surface. Sensors must be spaced close enough together so that local stress variations due to the ice failure process around a structure can be detected. The surface integral method is a useful technique for interpreting load and stress measurements since a knowledge of ice/structure interaction mechanisms is not needed. The accuracy of the method is determined by the density of sensor measurement



locations along the surface. A disadvantage of the technique is that a relatively large number of sensors are needed to determine the stress tensor along the surface of interest.

The surface integral method can be used to examine the effects of grounded ice rubble on structural ice loads. Two instrumented surfaces, one enclosing a structure and the other enclosing the structure and rubble field can be used to estimate the load acting only on the structure and also on the structure/rubble field system.

## INTRODUCTION

Resource exploration and establishing navigational aids in cold regions require bottom founded structures which can withstand the lateral forces generated by moving ice. Various methods have been used to estimate ice loads on structures including: mathematical analyses of ice interacting with structures (Croasdale, et al., 1977; Ralston, 1977, 1978; Kerr, 1978; and others), small scale model experiments (Edwards and Croasdale, 1977; Lewis and Croasdale, 1978), ice stress measurements in conjunction with mathematical descriptions of the stress field around structures (Strilchuk, 1977; Semeniuk, 1977) and direct circumferencial load and bending moment measurements on instrumented structures (Maattanen, 1977, 1980; Danys and Bercha, 1975). Mathematical analyses and small scale modeling techniques are currently the most used methods for estimating ice loads. Mathematical models are developed by making certain assumptions about the interaction mechanisms between a structure and ice, the rheology of ice and the environment which will be encountered. The usefulness of such models depends on how realistic the assumptions are. Small scale model tests and prototype tests (Robbins, et al., 1975; Verity, 1975) have been used to derive approximate empirical solutions for ice forces on structures. These solutions have then been compared with mathematical models (Kry, 1980). A drawback of both scale model tests and mathematical models is that the lack of load measurements for full size structures limits the extent to which modeling efforts can be compared and verified. The extreme cost and necessary overdesign of a full scale test structure has limited the number of direct load measurements available. Direct measurements of ice loads have in the past been conducted on relatively small diameter structures, lighthouses and pilings (Schwarz, 1970; Neill, 1970; Danys and Bercha, 1975; Maattanen,

1977). Recent efforts, however, have been directed at determining loads on full scale, exploratory structures (Strilchuk, 1977; Semeniuk, 1977; APOA, Dec. 1981). Determining the ice loads that act on full-scale structures provides important information to the designer including:

1. data to which scale model experiments and mathematical models can be compared,
2. lower bounds to possible ice forces,
3. an historical data base which, in conjunction with other environmental parameters, can be used for developing design criteria in probabilistic terms, and
4. information about the influence of ice rubble piles, which surround a structure, on ice forces.

The requirements for directly determining ice loads on structures from in-situ measurements have been discussed only briefly in the literature. Essentially two general methods are used. The first method involves measuring the forces acting on a structure using instruments attached to the structure. These can consist of either circumferential load cell or bending moment measurements and in this paper are referred to as load measurements (instrumented island). Load cells, sensitive to normal forces, are typically placed around the circumference of a structure and the load across the face of the structure is measured (Schwartz, 1970; Maattanen, 1977). Bending moment measurements of a structure have been used to determine ice loading but are difficult to interpret due to low signal output (Maattanen, 1977). Instrumented structures are an attractive method for determining ice forces since a knowledge of ice/structure interaction mechanisms is not required. However, unreliable load estimates can result from the use of sensors which do not respond to the total traction force.

Sensors which respond only to the normal component of load without regard for possible contributions from shear loads have been deployed on instrumented structures in the past. (Danys and Bercha, 1975; Maataanen, 1977). These can result in an underestimate of structural loading.

Recent efforts have been made to measure the total traction force on support members of the Yukon River bridge in Alaska and a bridge over the Ottauquechee River in Vermont (Burdick, personal communication; Sodhi, personal communication). Sensors are used that respond to both normal and shear loads.

A second method of determining structural ice loads involves imbedding sensors that respond to stress, in the ice around a structure and is referred to in this paper as stress measurements (in ice). The resultant ice forces are determined from stress measurements by using a mathematical model describing ice/structure interaction (Strilchuk, 1977; Templeton, 1979; Metge, et al., 1981). The disadvantages of using stress measurements in conjunction with mathematical models are the uncertainty in accuracy of stress measurements, uncertainty about the accuracy of the mathematical description for ice/structure interaction, and the uncertainty involved with using local ice stresses to calculate total ice forces. Kry (1978, 1979) has suggested that ice can fail in independent zones across wide structures. Such failure may result in local ice stresses which are not representative of the average stress acting on the structure. Ice stress measurements by Strilchuk (1977) provide an example of the difficulty of using local stress measurements to determine the total structural ice force.

Past difficulties of obtaining accurate ice load measurements for structures illustrates the importance of understanding what forces need to be measured and how in-situ measurements should be interpreted. This paper examines the questions associated with the deployment of stress (in ice) and load (instrumented island) sensors and the use of the resulting data to determine structural ice loads. Two methods for determining structural ice forces from in-situ measurements are described. The first method uses a mathematical description of ice/structure interaction and has been used previously. The second method uses a surface integral approach to compute total ice forces from in-situ measurements. This method is useful because it does not require a knowledge of ice/structure mechanisms and can be applied to any structural geometry. The surface integral method is then used to demonstrate how stress and load measurements can be used to determine the influence of ice rubble piles, which surround a structure, on ice forces.

### Calculating Ice Loads

#### Stress Measurements in Conjunction with a Mathematical Model of Ice/Structure Interaction

In-situ measurement of ice stresses around offshore structures is a difficult and time consuming task. The use of a mathematical model to describe ice/structure interaction can reduce the number of stress measurements needed to determine ice loads on a structure, provided that the model is realistic. To date the mathematical models describing ice/structure interactions that are used with stress sensor measurements have been for cylindrical structures. This geometry lends itself to a relatively simple solution as compared to more complicated shapes. The general assumption regarding

ice/structure interaction is that an elastic ice sheet moves past a cylindrical structure and the total force of the ice sheet is resisted by the structure (Figure 1). This situation has been described mathematically by Strilchuk (1977) and Wang (1978) and is given by

- (1)  $\sigma_r = N_r P \cos \theta$
- (2)  $\sigma_\theta = N_\theta P \cos \theta$
- (3)  $\tau_{r\theta} = N_{r\theta} P \sin \theta$  for  $-\pi/2 < \theta < \pi/2$ , and  
 $\sigma_r = \sigma_\theta = \tau_{r\theta} = 0$   
for  $\pi/2 < \theta < 3\pi/2$ .

The radial, tangential and shear stresses acting in the ice sheet in polar coordinates are respectively  $\sigma_r$ ,  $\sigma_\theta$  and  $\tau_{r\theta}$ . The average pressure acting on the structure along the diameter d-d' is P (Figure 1). The three coefficients  $N_r$ ,  $N_\theta$  and  $N_{r\theta}$  depend on the boundary condition at the ice/structure interface and are given by

- (4)  $N_r = 4/(\pi X)$  Strilchuk (1977), and  
for a fixed boundary condition along the ice-structure interface
- (5)  $N_r = 1/\pi ((1 + \nu)/X^3 - (3 + \nu)/X)$ ,
- (6)  $N_\theta = 1/\pi (-(1 + \nu)/X^3 + (1 - \nu)/X)$ ,
- (7)  $N_{r\theta} = 1/\pi ((1 + \nu)/X^3 + (1 - \nu)/X)$ , and  
for a frictionless boundary condition
- (8)  $N_r = -1/\pi ((1 - \nu)/X^3 + (3 + \nu)/X)$ ,
- (9)  $N_\theta = 1/\pi ((1 - \nu)/X^3 + (1 - \nu)/X)$ ,
- (10)  $N_{r\theta} = 1/\pi (-(1 - \nu)/X^3 + (1 - \nu)/X)$ ,

where  $X = r/R$  and  $\nu$  is Poisson's ratio (Wang, 1978). The radial distance from the center of the structure to a point in the ice sheet is given by  $r$  and the structure radius by  $R$ .

In application of the model the radius  $R$  is taken as the ice failure boundary. This could be the structure radius or the radius of a frozen annulus of ice around the structure. Ice movement measurements have been used to determine the direction of ice sheet motion and estimate the principal stress direction (Strilchuck, 1977). The average load on the structure can be estimated from a stress sensor, which measures stress in the radial direction, located in the region  $-\pi/2 < \theta < \pi/2$  (Figure 1). For example, the average stress acting on the island can be computed from

$$(11) \quad P = \sigma_r' / (N_r \cos \alpha),$$

where  $\sigma_r'$  is the stress reading at  $SS_1$  in

Figure 1 and  $\alpha$ , the angle between the stress sensor and Principal force direction is determined from ice movement measurements (Strilchuk, 1977).

A second method of estimating  $P$  without using ice movement measurements makes use of an array of four stress sensors,  $SS_1$  through  $SS_4$ , with angular spacings of  $\pi/2$  around a structure (Figure 1). Using equation (1) it is easy to show that

$$(12) \quad \alpha = \tan^{-1} (\sigma_r'' / \sigma_r')$$

$$(13) \quad P = \sigma_r' / (N_r \cos \alpha)$$

where

$$\sigma_r'' < \sigma_r', \sigma_r''' = \sigma_r'''' = 0 \text{ and}$$

$\alpha$  is the angle between the principal force direction and  $\sigma_r'$ . The stress readings at  $SS_1$ ,  $SS_2$ ,  $SS_3$  and  $SS_4$  in Figure 1 are respectively  $\sigma_r'$ ,  $\sigma_r''$ ,  $\sigma_r'''$  and  $\sigma_r''''$ . A four sensor technique has been discussed by Templeton (1979) although the mathematical formulation was not given.

It is unlikely that the ice/structure interaction model described by equations (1), (2) and (3) is adequate to determine structural ice loads from stress sensor measurements. The mechanical properties of ice and the failure mechanisms for ice around wide structures are complex and, in general, not well understood. Ice stress measurements around three exploration islands in the Canadian Beaufort Sea illustrate the difficulties of using a mathematical model to interpret stress measurements. Several different sensors around the islands were used to independently calculate the average pressure acting on the islands,  $P$ . The average pressure was found to vary significantly depending on the sensor and its location. In some cases the variations in  $P$  were greater than 690 KPa (100 PSI) for different sensor measurement locations (Strilchuk, 1977; Semuniuk, 1977).

The use of mathematical models to interpret stress measurements is probably even less reliable for structures with noncircular geometries (for example rectangular or polygonal shapes). The stress distribution around such structures can be complex and dependent on the direction of loading. Therefore, it is important to develop methods for interpreting stress and load measurements which do not require an understanding of the details of ice/structure interaction.

#### The Surface Integral Method for Calculating Structural Loads from In-Situ Measurements

A method for interpreting in-situ stress and load measurements that does not require an understanding of ice/structure interaction can be developed from first principals of continuum mechanics. In the analysis of structural loading only the surface forces or stresses due to the ice acting against a structure are important (body forces can be neglected). The surface force acting on an imagined surface in the interior of a



body is the stress vector of Euler and Cauchy's stress principle. According to this concept, the total force acting upon the region interior to a closed surface  $s$  is

$$(14) \quad \vec{F} = \oint_s \vec{T} ds,$$

where  $\vec{T}$  is the stress vector acting on the surface element  $ds$  whose outer normal vector is  $\vec{u}$  (Figure 2). A structure's geometry and the surface of integration,  $s$ , can be any shape. However, for the purposes of illustration a cylindrical structure and cylindrical surfaces are used in this paper. Figure 2 illustrates the concept of applying the surface integral method for calculating the load on a cylindrical structure. The surface  $s$  can be placed anywhere in the ice sheet provided that it encompasses the structure. Two possible surfaces of integration shown in Figure 2 include one surface that follows the circumference of the structure,  $S_1$ , and another in the ice sheet,  $S_2$ . Once the traction vector (stress vector) is known, the load acting on the body interior to  $s$  can be determined. The surface traction vector is defined as

$$(15) \quad \vec{T} = \vec{u} \cdot \{\Sigma\}.$$

where the stress tensor of the ice sheet is  $\{\Sigma\}$ . The stress tensor can be developed in several ways depending on the coordinate system. For the cylindrical structure depicted in Figure 2 in cartesian coordinates

$$(16) \quad \{\Sigma\} = [\sigma_{ij}] = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \text{ and } \vec{u} = \cos \theta \vec{i} + \sin \theta \vec{j},$$

where  $\vec{i}$  and  $\vec{j}$  are outward pointing unit vectors for the cartesian system.

The z component stresses are assumed to be negligible  $\tau_{xz} = \tau_{yz} = \sigma_z = 0$ .

The traction vector acting on ds is then

$$(17) \quad \vec{T} = \sigma_{ij} \vec{e}_j = (\sigma_x \cos\theta + \tau_{xy} \sin\theta) \vec{i} + (\sigma_y \sin\theta + \tau_{xy} \cos\theta) \vec{j},$$

where

$$(18) \quad ds = r \, d\theta \, dz.$$

Equation (17) shows that both normal and shear stresses contribute to  $\vec{T}$ . This means that load and stress measurements that are sensitive only to normal loading will cause structural loads to be underestimated as suggested above. It is also evident that three component stress sensor stations must be used at each measurement location in order to resolve the stress tensor along s.

An example calculation using Wang's (1978) solutions will be used to demonstrate the technique and to illustrate the importance of considering shear loads. Wang's solutions are presented in polar coordinates for which, with the aid of a coordinate transformation, equation (17) takes the form

$$(19) \quad \vec{T} = (\sigma_r \cos\theta - \tau_{r\theta} \sin\theta) \vec{i} + (\sigma_r \sin\theta + \tau_{r\theta} \cos\theta) \vec{j}.$$

The force on the cylindrical structure can now be computed for both the fixed and free boundary conditions using equation (14) and assuming that the direction of loading is colinear with the x axis (Figure 2).

$$(20) \quad \vec{F} = \int_0^t \int_0^{2\pi} \vec{T} r \, d\theta \, dz.$$

Substituting equations (1) and (3) in (19) and integrating equation (20) gives

$$(21) \quad \vec{F} = P t \pi r / 2 (N_r - N_{r\theta}) \vec{i}.$$

for the fixed boundary condition

$$F = P_{tr}/2 \left[ \underbrace{((1+\nu)/x^3 - (3+\nu)/x)}_{\text{radial component}} - \underbrace{((1+\nu)/x^3 + (1-\nu)/x)}_{\text{shear component}} \right] \vec{i} = -2P_{tr} \vec{i}$$

and for the frictionless boundary condition

$$F = P_{tr}/2 \left[ -\underbrace{((1-\nu)/x^3 + (3+\nu)/x)}_{\text{radial component}} - \underbrace{(-(1-\nu)/x^3 + (1-\nu)/x)}_{\text{shear component}} \right] \vec{i} = -2P_{tr} \vec{i}$$

The relative contribution of the radial and shear components can be shown for the special case where  $r = R$ . In this case the force for fixed boundary is

$$F = P_{tr} \left[ \underbrace{-1}_{\text{radial component}} \underbrace{-1}_{\text{shear component}} \right] \vec{i}$$

and for the frictionless boundary

$$F = P_{tr} \left[ \underbrace{-2}_{\text{radial component}} \underbrace{-0}_{\text{shear component}} \right] \vec{i}$$

In this example shear stress loading contributes from 0% to 50% of the total load for the structure depending on the boundary condition. This result demonstrates the importance of including shear force loading measurements in all but the simplest of loading situations (for example, loading perpendicular to one face of a rectangular structure).

In the surface integral formulation  $\{\Sigma\}$  is continuous along  $s$ . However, in practice  $\{\Sigma\}$  will be determined only at a finite number of locations around a structure with equation (14) to be solved numerically. The accuracy of the surface integral method will thus depend directly on the accuracy of load/stress measurements, the density of measurement locations along  $s$ , and the interpolation scheme used in numerically integrating equation (14). The density of measurement locations needed to adequately determine structural loading can depend on a number of factors including; the variability of the principal ice movement direction around a structure, the geometry of the structure, and the size of any independent failure zones across the structure. If measurement locations are too widely spaced, then local effects which may significantly affect the loading could be missed resulting in inaccurate load estimations.

Two possible deployment schemes for sensors would use the structure geometry to determine  $s$  (Figure 2). The first deployment, along  $s_1$ , consists of attaching load sensors, sensitive to both normal and shear loading, to a structure around its circumference. A second deployment consists of placing stress sensor arrays, an array is composed of three sensors oriented so that the principal directions can be determined, in the ice along  $s_2$  (Figure 2). Either deployment method can be used to determine the ice loads acting on the region interior to the surface. If grounded ice rubble is in the vicinity of the structure then only the first deployment method will yield reasonable structural load estimates. This results because locally grounded ice features can influence the magnitude of load transmitted to a structure (Kry, 1977). The effect can be examined by using both of the deployment methods described above. Measurements from the instrumented surface  $s_1$  along the ice/structure interface permit calculation of the

ice loading on the structure. A second instrumented surface,  $s_2$  enclosing both the structure and rubble field provides data for calculation of the ice loading on the structure/rubble field system. The load resistance and stress amplification characteristics of the rubble can then be determined by comparison.

### Summary and Conclusions

Estimates of ice loads on structures have been obtained by the use of mathematical analyses, small scale and prototype model tests, and load measurements on full-scale structures. Measurements of ice loads on full-scale structures are needed to provide (1) data for comparison with the results of the mathematical analyses and scale model tests, (2) lower bound estimates for ice forces, (3) a data base of loading events, and (4) to examine the influence of grounded ice features on structural ice loads. Methods of taking the data include attaching instruments to a structure to measure ice loads, and measuring stress using sensors embedded in an ice sheet around a structure. Mathematical models describing ice/structure interaction are used to interpret the stress measurements and to calculate ice loads on structures; no method has been described in the literature for interpreting load measurements. The advantage of using a mathematical model for interpreting stress measurements is that only a few sensors are needed to develop an estimate of ice loads on structures. However, the usefulness of the interpretation may be limited by the accuracy of the mathematical description and the uncertainty of using local ice stress measurements to calculate total ice forces.

A method of interpreting load and stress measurements using surface integrals was described in this paper. This uses the concept that the total force acting on a structure can be determined by summing the stress

vectors acting on an imaginary surface that encompasses the structure.

Application of the surface integral method requires that the normal and shear components of load or stress be known along the surface. This means that sensor arrays, capable of resolving normal and shear loads, must be placed along the surface. The spacing between sensor arrays should be small enough so that local stress or load changes due to the ice failure process around a structure can be detected. The surface integral method is an attractive technique for interpreting load and stress measurements since a knowledge of ice/structure interaction mechanisms is not required. The primary disadvantage of the technique is that a relatively large number of sensors are needed to adequately determine the stress tensor along the surface of interest.

The surface integral method can be used to examine the effects of a grounded rubble field on structural ice loading. One instrumented surface along an ice/structure boundary would be used to determine the load acting on the structure. A second instrumented surface enclosing both the structure and rubble field could then be used to estimate the load acting on the combined structure/rubble field system.

## REFERENCES

- APOA, 1981, Profile: Ken Croasdale, Ice Engineer, APOA Review, Vol. 4, No. 3, pp. 16-17.
- Croasdale, K. R., Morgenstern, N. R. and Nuttal, J. B., 1977. Indentation tests to investigate ice pressures on vertical piers. J. of Glaciology, 19(81), pp. 301-312.
- Danys, J. V. and Bercha, F. G., 1975. Determination of ice forces on a conical offshore structure. Proceedings of the Third International Conference on Port and Ocean Engineering under Arctic Conditions, 11-15 August 1975, University of Alaska, pp. 741-751.
- Edwards, R. Y. and Croasdale, K. R., 1977. Model experiments to determine ice forces on conical structures. J. of Glaciology, 19(81), p. 660.
- Kerr, A. D., 1978. On the determination of horizontal forces a floating ice plate exerts on a structure. J. of Glaciology, 20(82), pp. 123-134.
- Kry, P. R., 1977. Ice rubble fields in the vicinity of artificial islands. Proceedings 4th International Conference on Port and Ocean Engineering under Arctic Conditions, St. John's Nfld., Vol. 1, pp. 200-211.
- Kry, P. R., 1978. A statistical prediction of effective ice crushing stresses on wide structures. Proceedings, International Association for Hydraulic Research, Symposium on Ice Problems, Lulea, Sweden, Part 1, pp. 33-47.
- Kry, P. R., 1979. Implications of structure width for design ice forces. International Union of Theoretical and Applied Mechanics, Symposium on the Physics and Mechanics of Ice. Copenhagen, Denmark, Aug. 6-10.
- Kry, P. R., 1980. Third Canadian Geotechnical Colloquium, Ice Forces on Wide Structures. Can. Geotech. J., 17, 97-113.

- Lewis, J. W. and Croasdale, K. R., 1978. Modeling the interaction between pressure ridges and conical structures. Part I, Proceedings of the IAHR Ice Symposium on Ice Problems August 7-9, 1978, pp. 165-196.
- Maattanen, M., 1977. Ice-force measurements at the Gulf of Bothnia by the instrumented Kemi 1 Lighthouse. Fourth Int. Conf. on POAC 1977, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Sept. 26-30, 1977.
- Maatanen, M., 1980. Ice-force measurements in Finish lighthouses. Proceedings of the Workshop on Sea Ice Field Measurements, St. John's, Apr. 29-May 1, 1980, pp. 343-353.
- Metge, M., Danielewicz, G. and Hoare, R., 1981. On measuring large-scale ice forces; Hans Island 1980. Proc. of the Sixth POAC, Quebec, Canada, July 27-31, 1981, pp. 629-642.
- Neill, C. R., 1970. Ice pressure on bridge piers in Alberta, Canada. Proceedings, IAHR, Symposium on Ice and Its Action on Hydraulic Structures, Reykjavik, Iceland, Contribution 6.1, 8 pp.
- Ralston, T. D., 1977. Ice force design considerations for conical offshore structures. Proceedings, 4th International Conference on Port and Ocean Engineering Under Arctic Conditions, St. John's, Newfoundland, Vol. II, pp. 741-752.
- Ralston, T. D., 1978. An analysis of ice sheet indentation. Proceedings International Association for Hydraulic Research Symposium on Ice Problems, Lulen, Sweden, Part I, pp. 13-31.
- Robbins, R. J., Verity, P. H., Taylor, T. P., and Metge, M., 1975. Techniques for the study of ice-structure interaction. Proceedings 3rd International Conference on Port and Ocean Engineering under Arctic Conditions, Fairbanks, Alaska, Vol. II, pp. 911-924.



- Semeniuk, A., 1977. Ice pressure measurements at Arnak L-30 and Kannerk G-42, Arctic Petroleum Operators Association Project 122, 331 p.
- Schwarz, I. J., 1970. The pressure of floating ice-fields on piles. Proceedings, International Association for Hydraulic Research, Symposium on Ice and Its Action on Hydraulic Structures, Reykjavik, Iceland, Contribution 6B, 12 pp.
- Strilchuk, A. R., 1977. Ice pressure measurements: Netserk F-40, 1975-76. Arctic Petroleum Operators Association Project 105, 279 pp.
- Templeton, J. S., III, 1979. Ice defense and monitoring. Technical Seminar on Alaska Beaufort Sea Gravel Island Design, Exxon Co., Oct. 1979, 26 pp.
- Verity, P. H., 1975. A.P.O.A. project 65 small prototype tests, winter 1973/74. Arctic Petroleum Operators Association, Report No. 65-1.
- Wang, Y. S., 1978. Buckling of a half ice sheet against a cylinder. Jour. of Eng. Mech. Div., Vol. 104, No. EM5, pp. 1131-1145.

## FIGURE CAPTIONS

Figure 1. Ice sheet moving past a cylindrical structure.

Figure 2. Ice sheet moving past a cylindrical structure. Two surfaces of integration are shown: one along the ice/structure interface and one in the ice sheet surrounding the structure.





